

Final Exam, MTH 320, SPRING 2009

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WORK OUT ONLY FOUR QUESTIONS

QUESTION 1. (i) Let $a, b \in$ a group $(M, *)$ such that $\mid b \mid= k < \infty$. Show that $\mid a * b * a^{-1} =\mid b \mid$.

(ii) Find a group homomorphism, say f, from $(Z_{10}, +_{10})$ into $(Z_5^*, \times_5)$ such that $Ker(f) \neq \{e\}$. Find image (f). Find Ker(f).

QUESTION 2. Given that H is a normal cyclic subgroup of $(M, *)$. Let K be a subgroup of H. Prove that K is also a normal subgroup of M. (We know that K is a subgroup of M, just show it is normal in M).

QUESTION 3. (i) We know that $6/8 + 3Z \in Q/3Z$. Find $| 6/8 + 3Z |$.

(ii) Give me an example of a group with 49 elements that is not cyclic .

(iii) Give me an example of two group (M, *) and (H, *) such that H is a normal subgroup of M and M/H is cyclic but M is not cyclic.

QUESTION 4. (i) Given (M, *) is a group and H is a subgroup of $Z(M)$ such that $(M/H, \wedge)$ is cyclic. Prove that $(M, *)$ is an abelian group.

(ii) Let Q^+ be the set of all nonzero positive rational numbers. We know that $(Q^+, \times)$ is a group. Let $f : (Z, +) \to (Q^+, \times)$ be a group homomorphism. Show that $f(a) = 1$ for every $a \in Z$

QUESTION 5. (i) Let $(M, *)$ be a a finite group and H be a normal subgroup of M. Given $a * H \in M/H$ such that $| a * H |= n < \infty$. Prove that M must have an element of order n.

(ii) Give me two elements, say a and b, in the group $[(Z_{33}, +_{33}) \oplus (Z_3^*, \times_3)]$ such that $| a |=| b |= 22$.

QUESTION 6. Let f be a group-homomorphism from (M, *) into $(H, \Box)$. Let K be a subgroup of H.

1) Show that $D = f^{-1}(K) = \{m \in M \mid f(m) \in K\}$ is a subgroup of M.

2) If K is also normal in H, then show that D in part (1) is also normal in M.

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