Krstin Raed

15/15 :)

Quiz I MTH 111, Spring 2019

Ayman Badawi

$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$CF^2 = 25 - 9$

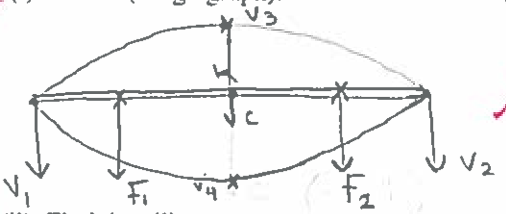
$= 16$

$\boxed{CF = 4}$

QUESTION 1. Consider the ellipse $\frac{(x+2)^2}{25} + \frac{(y-1)^2}{9} = 1$

$25 = \left(\frac{k}{2}\right)^2$, $9 = b^2$

2 (i) Sketch (rough graph).



$c = (-2, 1)$

2 (ii) Find the ellipse-constant, k

$\left(\frac{k}{2}\right)^2 = 25 \qquad \frac{k}{2} = \sqrt{25} \rightarrow k = 5 \times 2 \rightarrow \boxed{k = 10}$

$b^2 = 9$

$\boxed{b = 3}$

2 (iii) Find all 4 vertices

$V_1\ (-2-5, 1)\ \ (-7, 1)$

$V_2\ (-2+5, 1)\ \ (3, 1)$

$V_3\ (-2, 4)$

$V_4\ (-2, -2)$

2 (iv) Find the Foci

$F_2\ (-2+4, 1)$

$(2, 1)$

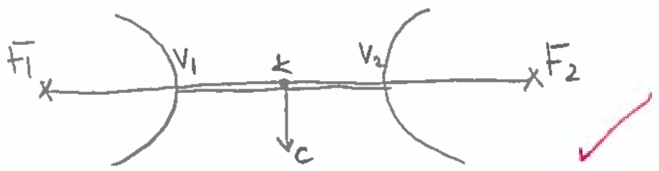
$F_1\ (-2-4, 1)$

$(-6, 1)$

QUESTION 2. Consider the hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{16} = 1.$

2 (i) Sketch (rough graph).

$c = (3, -2)$



1 (ii) Find the heperbola-constant, k

$\left(\frac{k}{2}\right)^2 = 9 \qquad \frac{k}{2} = \sqrt{9} \qquad \boxed{k = 3 \times 2 = 6}$

2 (iii) Find all vertices

$V_1\ (3-3, -2)$

$(0, -2)$

$V_2\ (3+3, -2)$

$(6, -2)$

2 (iv) Find the Foci

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

$CF^2 = 9 + 16$

$CF^2 = 25$

$\boxed{CF = 5}$

$F_1\ (3-5, -2)$

$(-2, -2)$

$F_2\ (3+5, -2)$

$(8, -2)$

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Quiz II MTH 111, Spring 2019

Ayman Badawi

15/15 :)

QUESTION 1. Consider the parabola $y = 3x^2 - 6x + 2$

3 (i) Write the equation above in the standard form.

$y = 3x^2 - 6x + 2$

$y = (3(x^2 - 2x)) + 2$

$3((x-1)^2 - 1^2) + 2$

$y = 3(x-1)^2 - 3 + 2$

$y = 3(x-1)^2 - 1$

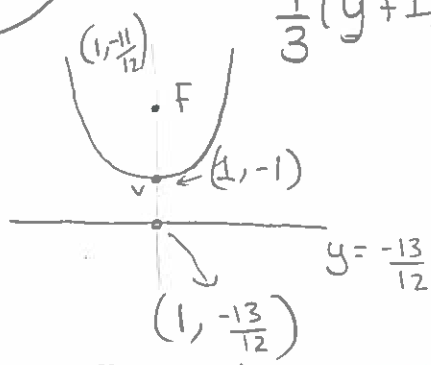
$\frac{(y+1)}{3} = \frac{3(x-1)^2}{3}$

$\frac{1}{3}(y+1) = (x-1)^2$

2 (ii) Sketch the graph (roughly)

y = so up or down

$4d = \frac{1}{3}$ $d = \frac{1}{12}$ so up



1 (iii) Find the vertex.

$\boxed{(1,-1)}$

1 (iv) Find the FOCUS.

$F = \left(1, -1 + \frac{1}{12}\right)$ $\boxed{\left(1, \frac{-11}{12}\right)}$

1 (v) Find the equation of the directrix line

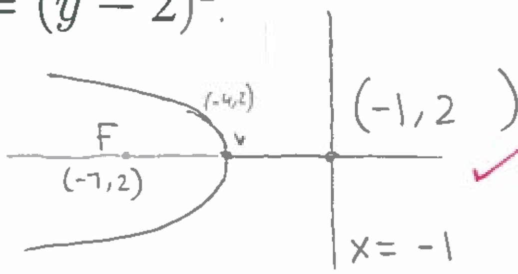
$\boxed{y = \frac{-13}{12}}$

QUESTION 2. Consider the parabola $-12(x+4) = (y-2)^2$.

4 (i) Sketch (rough graph).

$4d = -12$ (x so its right or left)

$\boxed{d = -3}$ negative so left



1 (ii) Find the focus $(-4-(3), 2)$ $\boxed{(-7, 2)}$

1 (iii) Find the vertex

$\boxed{(-4,2)}$

1 (iv) Find the equation of the directrix line.

$\boxed{x = -1}$

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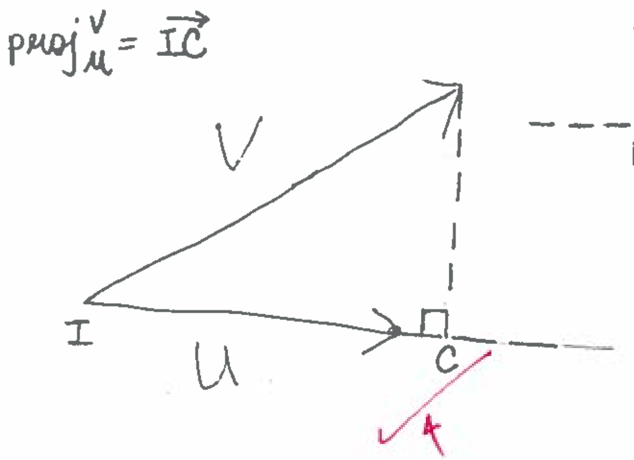
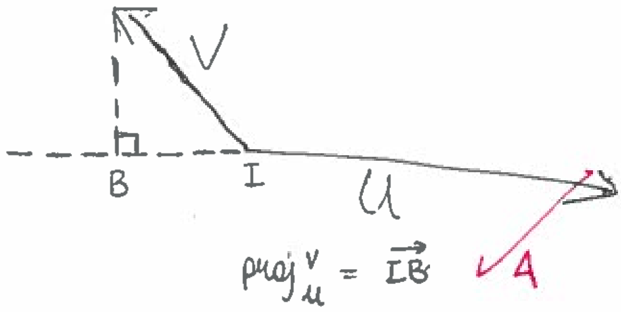
Draw the projection V over U

15/15





4

4

Let $U = \langle 2, 2 \rangle$, $V = \langle -3, 4 \rangle$

$u \cdot v = -6 + 8 = 2$; $|u| = \sqrt{(2)^2 + (2)^2} = \sqrt{8} = 2\sqrt{2}$

Find $proj_u^V = \left(\frac{u \cdot v}{|u|^2}\right) u = \frac{\cancel{2}}{\cancel{8}\,4}\langle 2,2 \rangle = \frac{1}{4}\langle 2,2 \rangle = \langle \frac{1}{2}, \frac{1}{2} \rangle$

4

Find $\left| proj_u^V \right| = \frac{|u \cdot v|}{|u|} = \frac{2}{\sqrt{8}} = \frac{\cancel{2}}{\cancel{2}\sqrt{2}} = \frac{1}{\sqrt{2}}$

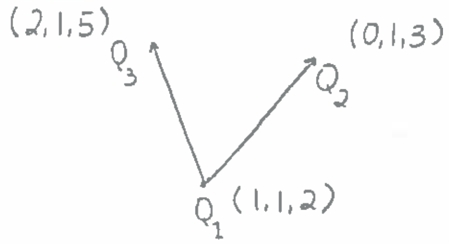
3

Quiz V MTH 111, Spring 2019

Ayman Badawi

15/15 :)

7/7 **QUESTION 1.** Let $Q_1 = (1,1,2), Q_2 = (0,1,3), Q_3 = (2,1,5)$. Find the equation of the plane that passes through $Q_1, Q_2, Q3$.



$N \perp$ Plane

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\overrightarrow{Q_1Q_2} = \langle 0-1, 1-1, 3-2 \rangle = \langle -1, 0, 1 \rangle$ ✓ 1

$\overrightarrow{Q_1Q_3} = \langle 2-1, 1-1, 5-2 \rangle = \langle 1, 0, 3 \rangle$ ✓ 1

$\overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3} = \begin{vmatrix} i & j & k \\ -1 & 0 & 1 \\ 1 & 0 & 3 \end{vmatrix} = (\ 0\ )i - (-3-1)j + (\ 0\ )k = 4j = \langle 0, 4, 0 \rangle$ ✓ 3

choose Q_1 & a random point

$w = (x, y, z)$

$Q_1 = (1, 1, 2)$

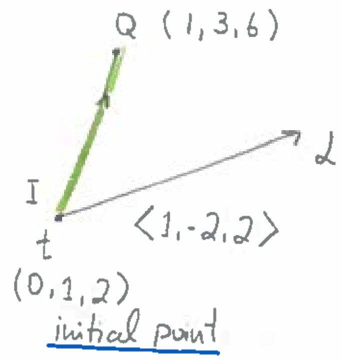
$\overrightarrow{Q_1w} = (x-1, y-1, z-2)$

$N \cdot \overrightarrow{Q_1w} = \langle 0, 4, 0 \rangle \cdot \langle x-1, y-1, z-2 \rangle = 0$

$0(x-1) + 4(y-1) + 0(z-2) = 0$

$4(y-1) = 0$ ✓ 2

8/8 **QUESTION 2.** Let $L : x = t, y = -2t + 1, z = 2t + 2(\ t \in R)$, and $Q = (1,3,6)$. Find $|QL|$.



$\left.\begin{array}{l} x = t \\ y = -2t+1 \\ z = 2t+2 \end{array}\right\} t \in \mathbb{R}$

$(1, 0, 2) = I$

$\langle 1, -2, 2 \rangle = D$ ✓ 1

$|D| = \sqrt{(1)^2 + (-2)^2 + (2)^2} = \sqrt{1+4+4} = \sqrt{9} = 3$

$\overrightarrow{IQ} = \langle 1-0, 3-1, 6-2 \rangle = \langle 1, 2, 4 \rangle$ ✓ 1

$\overrightarrow{IQ} \times \vec{D} = \begin{vmatrix} i & j & k \\ 1 & 2 & 4 \\ 1 & -2 & 2 \end{vmatrix}$

$= (4-(-8))i - (2-4)j + (-2-2)k$

$= (4+8)i - (-2)j + (-4)k$

$= 12i + 2j - 4k$

$\langle 12, 2, -4 \rangle$ ✓ 3 $= \sqrt{(12)^2 + (2)^2 + (-4)^2}$ ✓

$= \sqrt{144+4+16} = \sqrt{164} = 2\sqrt{41}$ ✓

$|QL| = \dfrac{|\vec{D} \times \overrightarrow{IQ}|}{|D|} = \dfrac{2\sqrt{41}}{3}$ ✓ 3

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Quiz 6 MTH 111, Spring 2019

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15/15 :)

Ayman Badawi

QUESTION 1. Find $f'(x)$ and DO NOT SIMPLIFY

2 a) $f(x) = 10(3x^4 + 12x^3 - 10x + 5)^{11}$

$= 10 \cdot 11\,(3x^4 + 12x^3 - 10x + 5)^{10} \cdot (12x^3 + 36x^2 - 10)$

3 b) $f(x) = \sqrt[5]{x^2} + \frac{12}{x^{10}} + 7x - 3$

$= x^{2/5} + 12x^{-10} + 7x - 3$

$= \frac{2}{5}x^{-3/5} - 120x^{-11} + 7$

8/8

3 c) Given $k(x) = f(2x^2 + x - 16)$. Find $k'(3)$ if $f'(5) = -7$.

$k'(x) = f'(2x^2 + x - 16) \cdot (4x + 1)$

$k'(3) = f'(2 \times 9 + 3 - 16) \cdot (12 + 1)$

$= f'(5) \cdot 13$

$= -7 \cdot 13 = -91$

QUESTION 2. Let $f(x) = x^3 - 6x^2 - 15x + 1$.

3 a) Find the sign of $f'(x)$.

$f'(x) = 3x^2 - 12x - 15$; For critical value: $f'(x) = 0$

P = −5
S = −4
−5 × 1

$\Rightarrow 0 = 3x^2 - 12x - 15$

$= 3(x^2 - 4x - 5)$; $\Rightarrow 0 = x^2 - 4x - 5 \Rightarrow 0 = (x-5)(x+1)$

as $3 \neq 0$

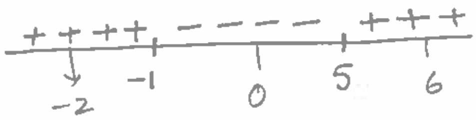
$\Rightarrow x - 5 = 0$ OR $x + 1 = 0 \Rightarrow x = 5$ OR $x = -1$

sign of $f'(x)$:

At $x = -2$; $f'(-2) = 12 + 24 - 15 = 21 > 0$

At $x = 0$; $f'(0) = -15 < 0$

At $x = 6$; $f'(6) = 3 \times 36 - 72 - 15 = 21 > 0$



1 b) By staring at (a) find the critical values.

$x = 5$ or $x = -1$

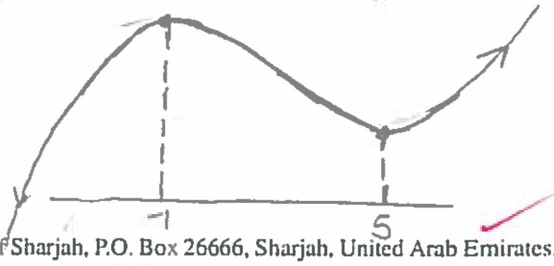
7/7

2 c) By staring at (a), for what values of x does $f(x)$ increase (decrease)?

$\therefore f(x)$ increases : $(-\infty, -1) \cup (5, \infty)$

$f(x)$ decrease : $(-1, 5)$

1 d) By staring at (a), sketch $f(x)$ (roughly).



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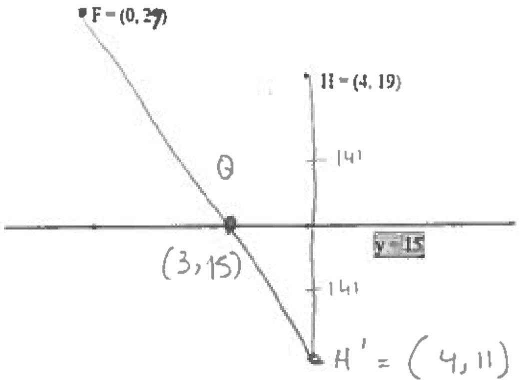
Quiz 7 MTH 111, Spring 2019

Ayman Badawi

15/15 :)

QUESTION 1. Given $H = (4, 19)$, $F = (0, 27)$. Find a point on the line $y = 15$, say Q, such $|FQ| + |QH|$ is minimum.



$Q = (3, 15)$

$y = mx + b$

$m = \frac{27 - 11}{0 - 4} = -4$

$y = -4x + b$

$11 = -4(4) + b$

$b = 27$

$y = -4x + 27$

$Q = (x, 15)$

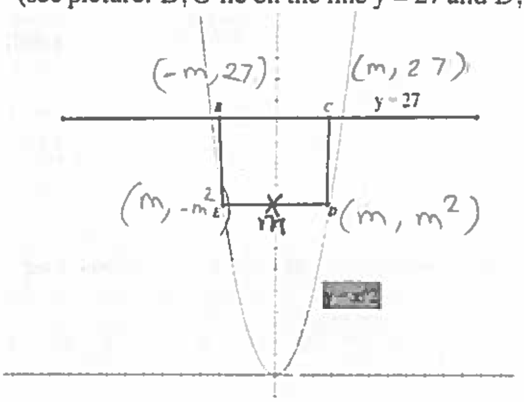
$15 = -4x + 27$

$x = \frac{15 - 27}{-4}$

$= 3$

8/8

QUESTION 2. Consider the following picture. We need to construct a rectangle B, C, D, E with maximum area between $y = 27$ and $y = x^2$ (see picture: B, C lie on the line y = 27 and D, E lie are the curve $y = x^2$. Also note that $mE = mD$).



Find $|BC|, |CD|$.

$A = 2(3) \cdot (27 - 3^2)$

$= 108 \text{ units}^2$

$|BC| = 2m$

$= 6 \text{ units}$

$|CD| = 27 - 3^2$

$= 18 \text{ units}$

$A = |cd||ED|$

$A = 2m \cdot (27 - m^2)$

$A = 54m - 2m^3$

$A' = 54 - 6m^2$

$0 = 54 - 6m^2$

$m^2 = -54/-6$

$= 9$

$m = \pm 3$

check test.

$A'' = -12m$

$-12(3) = - = \max$

$\therefore 3$ is m

7/7

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Quiz 7 MTH 111, Spring 2019

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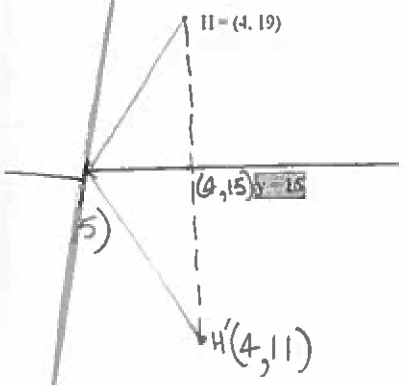
QUES[TION 1.] $H = (4, 19)$, $F = (0, 27)$. Find a point on the line $y = 15$, say Q, such $|FQ| + |QH|$ is minimum.



m of line $FH' = \frac{27-11}{0-4} = \frac{\cancel{16}\,4}{-\cancel{4}} = -4$

Eq of FH': $y = -4x + b$

Putting F in eq of FH' we get

$27 = -4(0) + b \Rightarrow b = 27$

now, Eq. of line FH' is

$y = -4x + 27$

Putting $y = 15$ we get:

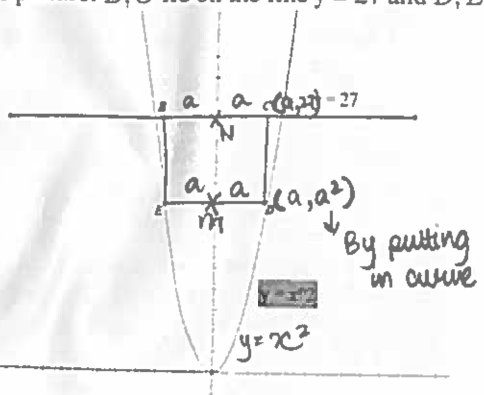
$15 - 27 = -4x$

$\Rightarrow \frac{+\cancel{12}\,3}{+\cancel{4}} = x \Rightarrow x = 3$

$\therefore$ req pt. $B = (3, 15)$

8/8

QUEST[IO]N 2. Consider the following picture. We need to construct a rectangle B, C, D, E with maximum area between $y = 27$ in $y = x^2$ (see picture: B, C lie on the line y = 27 and D, E lie are the curve $y = x^2$. Also note that $mE = mD$).



Find $|BC|, |CD|$.

let $ME = MD = a$

As Fig is symmetrical about y-axis

$BN = NC = a$

Area BCED, A is max.

$A = |BC||CD|$

now, From Fig:

$|BC| = 2a$

$|CD| = 27 - a^2$

$\therefore$ Area, $A = 2a(27 - a^2) = 54a - 2a^3$

$A' = 54 - 6a^2$

For critical values Let $A' = 0$; $0 = 54 - 6a^2 \Rightarrow 6a^2 = 54$

$a^2 = 9 \Rightarrow a = 3$ ($a \neq -3$ as length)

now, to check if $a = 3$ is max: $A'' = -12a$

$= -12(3) = -36 < 0 \Rightarrow$ max.

$\therefore$ at $a = 3$ Area BCDE is max.

$|BC| = 2(3) = 6$ units

$|CD| = 27 - (3)^2 = 27 - 9 = 18$ units

7/7

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