

Exam I: MTH 111, Spring 2019

$F = v \times w$

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$$\text{Points} = \frac{87}{87}$$

QUESTION 1. b) **(4 points)** Given $A = (6, 10)$, $B = (-7, 3)$, and $C = (-4, -2)$ are the vertices of a triangle. Find the area of the triangle ABC.

Area of the triangle ABC $= \frac{1}{2}|AB \times AC|$

$AB = \langle -13, -7 \rangle$ (B-A)

$AC = \langle -10, -12 \rangle$ (C-A)

$$AB \times AC = \begin{vmatrix} i & j & k \\ -13 & -7 & 0 \\ -10 & -12 & 0 \end{vmatrix} = 0i - 0j + \underline{86k} = \underline{86}$$

Area of $\Delta ABC = \frac{1}{2} 86 = \boxed{43 \text{ units}^2}$

c) **(3 points)** Find a vector F that is perpendicular to both vectors $V = \langle 2, 6, -3 \rangle$ and $W = \langle 5, -4, 1 \rangle$ such that $|F| = 111$.

$$F = v \times w = \begin{vmatrix} i & j & k \\ 2 & 6 & -3 \\ 5 & -4 & 1 \end{vmatrix} = -6i - 17j - 38k$$

$|F| = 111 \quad = \frac{111}{|F|} F$

$$= \boxed{\frac{111}{42} \langle -6, -17, -38 \rangle}$$

QUESTION 2. a) **(4 points)** The line $L_1 : x = -2t - 3, y = -3t + 3, z = 4t - 2$ ($t \in R$) intersects the line $L_2 : x = 2w - 13, y = 4w - 15, z = 4w - 6$ ($w \in R$) in a point Q. Find Q.

$L_1: x = -2t - 3$
$y = -3t + 3$
$z = 4t - 2$

$L_2: x = 2w - 13$
$y = 4w - 15$
$z = 4w - 6$

use substitution method

find pt of intersection: $-2t - 3 = 2w - 13$

$\frac{-2t}{-2} = \frac{2w - 13 + 3}{-2}$

$\boxed{t = -w + 5}$ sub in second equ

$-3(-w + 5) + 3 = 4w - 15$

$3w - 15 + 3 = 4w - 15$

$4w - 3w = -15 + 15 + 3$

$\boxed{1w = 3}$

$t = -3 + 5$

$\boxed{t = 2}$

- now sub in each line to get intersection pt

$-2(2) - 3 = 2(3) - 13$

$\boxed{-7 = -7}$

$-3(2) + 3 = 4(3) - 15$

$\boxed{-3 = -3}$

$4(2) - 2 = 4(3) - 6$

$\boxed{6 = 6}$

$$\boxed{\text{Intersection pt} = Q = (-7, -3, 6)}$$

b) **(2 points)** Are the lines in (a) perpendicular? Explain

$D_1 = \langle -2, -3, 4 \rangle$

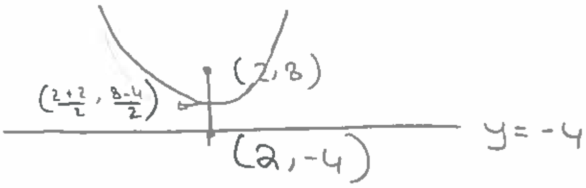
$D_2 = \langle 2, 4, 4 \rangle$

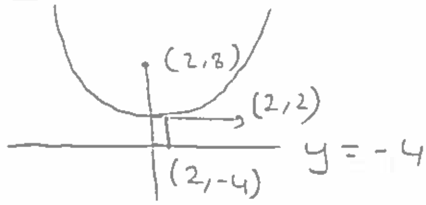
$D_1 \cdot D_2 = (-2 \times 2) + (-3 \times 4) + (4 \times 4)$
$= 0$

So they are perpendicular because their dot product is zero & they intersect

QUESTION 3. Given $y = -4$ is the directrix of a parabola that has the point $F = (2, 8)$ as its focus point.

a) **(2 points)** Roughly, sketch such parabola.





d = 6 & its up

4

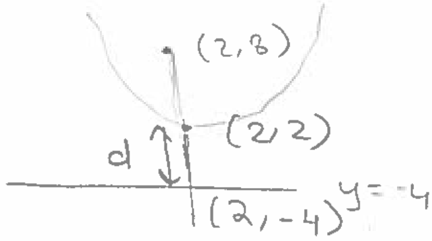
b)**(4 points)** Find the equation of the parabola

$4d(y-2) = (x-2)^2$

$4(6)(y-2) = (x-2)^2$

$24(y-2) = (x-2)^2$

$\boxed{d=6}$



$d = (-4-2$

-6

c) **(2 points)** Find the vertex of the parabola, say V.

$V = (2,2)$

QUESTION 4. Given $y = 4x^2 + 24x - 3$ is an equation of a parabola.

a)**(3 points)** Write the equation in the standard form.

$y = 4x^2 + 24x - 3$

$y = 4(x^2 + 6x) - 3$

$y = 4((x+3)^2 - 9) - 3$

$y = 4(x+3)^2 - 36 - 3$

$y = 4(x+3)^2 - 39$

$\frac{1}{4}(y+39) = \frac{4(x+3)^2}{4}$

$\frac{1}{4}(y+39) = (x+3)^2$

$4d = \frac{1}{4}$

$d = \frac{1}{4 \times 4}$

$\boxed{d = \frac{1}{16}}$

so +

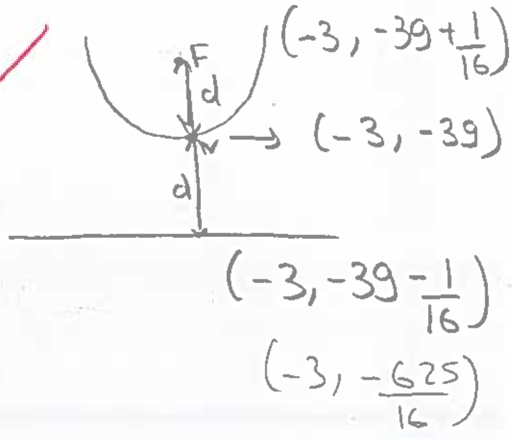
b) **(2 points)** Find the equation of the directrix line.

$y = -\frac{625}{16}$

c)**(2 points)** Find the focus, say F

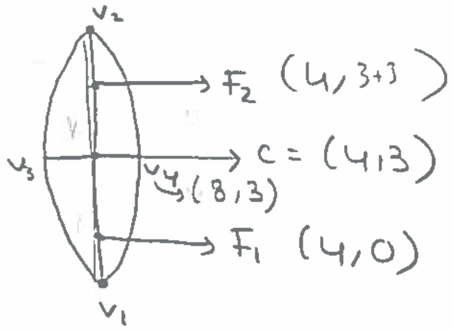
$F = (-3, -39 + \frac{1}{16}) = (-3, -\frac{623}{16})$

d)**(2 points)** Roughly, sketch the graph of such parabola.

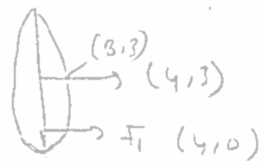


QUESTION 5. An ellipse is centered at $(4, 3)$, $F_1 = (4, 0)$ is one of the foci, and $(8, 3)$ is one of the vertices.

(i) **(2 points)** Roughly, sketch such ellipse.



x does not change



$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$

$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$

$25 = \left(\frac{k}{2}\right)^2$

$\boxed{CF = 3}$

$\boxed{b = 4}$

(ii) **(3 points)** Find the ellipse-constant K.

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2$$

$$3^2 = \left(\frac{k}{2}\right)^2 - 4^2$$

$$\boxed{k = 10}$$

(iii) **(2 points)** Find the second foci of the ellipse.

$$F_2 = (4, 3+3)$$
$$(4, 6)$$

(iv) **(3 points)** Find the remaining three vertices of the ellipse

$$V_1 = \left(4, 3 - \frac{10}{2}\right) \quad \boxed{(4, -2)}$$

$$V_2 \left(4, 3 + \frac{10}{2}\right) \quad (4, 8)$$

$$V_3 (0, 3)$$

(v) **(3 points)** Find the equation of the ellipse.

$$\frac{(y-3)^2}{\left(\frac{10}{2}\right)^2} + \frac{(x-4)^2}{4^2} = 1$$

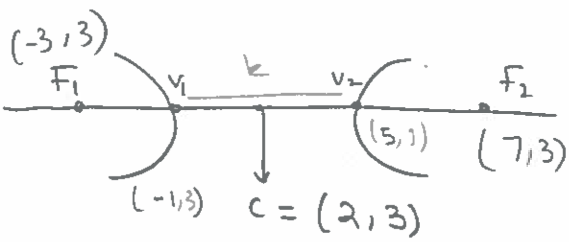
$$\frac{(y-3)^2}{25} + \frac{(x-4)^2}{16} = 1$$

QUESTION 6. Consider the hyperbola $\frac{(x-2)^2}{9} - \frac{(y-3)^2}{16} = 1.$

$\left(\frac{k}{2}\right)^2$ b^2

a) **(2 points)** Draw the hyperbola, roughly

under x so right left



b) **(2 points)** Find the hyperbola-constant K.

$$\left(\frac{k}{2}\right)^2 = 9$$

$$\frac{k}{2} = \sqrt{9}$$

$$k = 3 \times 2$$

$$\boxed{k = 6}$$

c) **(3 points)** Find the two vertices of the hyperbola.

$V_2 = (2+3, 3)$

$(5,3)$

$V_1 = (2-3, 3)$

$(-1,3)$

$CF^2 = \left(\frac{k}{2}\right)^2 + b^2$

d) **(3 points)** Find the foci of the hyperbola.

$F_1 = (2-5, 3)$ $(-3,3)$

$CF^2 = 9 + 16$

$= 25$

$F_2 = (2+5, 3)$ $(7,3)$

$\boxed{CF = 5}$

QUESTION 7. (4 points) Given two lines $L_1 : x = t + 1, y = 2t + 4, z = -5t + 3$ ($t \in R$) and $L_2 : x = 2w - 1, y = 4w + 1, z = -10w + 13$ ($w \in R$). Is L_1 parallel to L_2? Explain (show the work)

• 2 lines are // If they have cst & they do not intersect

L_1 : $x = t+1$
$y = 2t+4$
$z = -5t+3$

L_2 : $x = 2w-1$
$y = 4w+1$
$z = -10w+13$

take $t = 0$

$1 = 2w - 1$
$4 = 4w + 1$
$3 = -10w + 13$

$2w = 2$, $w = 1$
$4w = 4-1$, $w = \frac{3}{4}$
$10w = 13-3$
$10w = 10$
$w = 1$

$D_1 \langle 1, 2, -5 \rangle$ $D_2 \langle 2, 4, -10 \rangle$

$1 = c\,2$ $c = \frac{1}{2}$
$2 = c\,4$ $c = \frac{1}{2}$
$-5 = c(-10)$ $c = \frac{1}{2}$

they have a cst

$2w = 2$
$\boxed{w = 1}$
$4 - 1 = 4w$
$3 = 4w$
$\boxed{w = \frac{3}{4}}$
$3 - 13 = -10w$
$-10 = -10w$

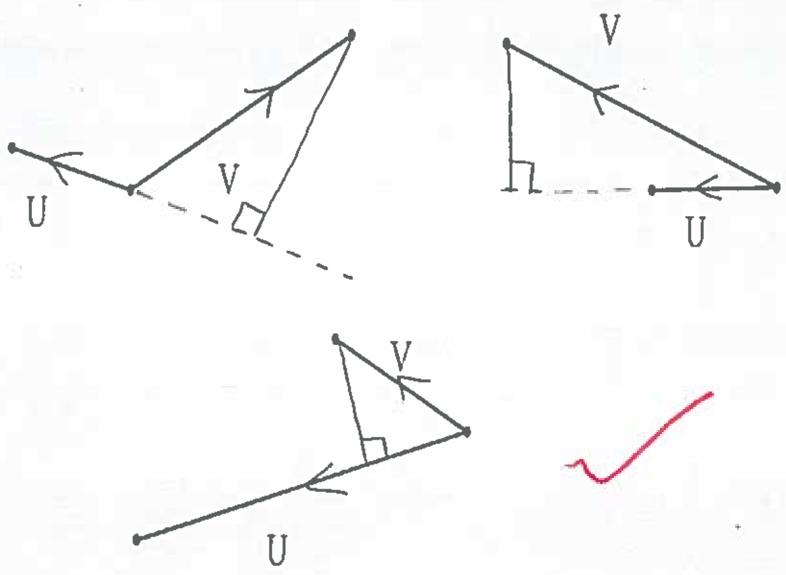
they do not intersect

$L_1 // L_2$

QUESTION 8. (6 points)

$proj_U^V$

Stare at the below. Then find Projection of V over U



QUESTION 9. (4 points) Find the equation of the plane that contains the points $Q_1 = (4, 4, 0)$. $Q_2 = (0, 2, 6)$ and $Q_3 = (4, 0, 8)$.

$N = \overrightarrow{Q_1Q_2} \times \overrightarrow{Q_1Q_3}$

$\langle -4, -2, 6\rangle \times \langle 0, -4, 8\rangle$

$$\begin{vmatrix} i & j & k \\ -4 & -2 & 6 \\ 0 & -4 & 8 \end{vmatrix} = 8i + 32j + 16k$$

$\langle 8, 32, 16\rangle$

choose a pt

$Q_1 = (4, 4, 0)$

$\boxed{8(x-4) + 32(y-4) + 16(z-0) = 0}$

$\boxed{8(x-4) + 32(y-4) + 16z = 0}$

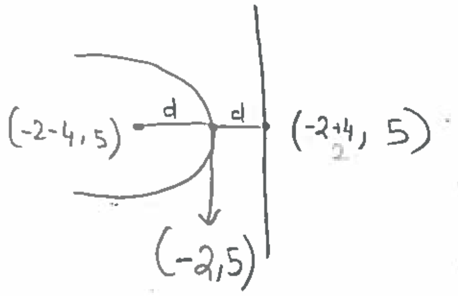
QUESTION 10. (6 points) Consider the parabola $-16(x+2) = (y-5)^2$.

(i) Sketch the parabola

$4d = -16$

$d = -4$

& before x so its left



(ii) Find the equation of the directrix line

$X = -2 + 4$

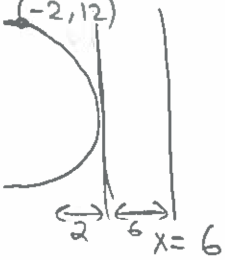
$\boxed{X = 2}$

(iii) Find the focus point.

Focus $= (-2-4, 5)$

$(-6, 5)$

QUESTION 11. (4 points) Given that $x = 6$ is the directrix line of a parabola that has F as its focus point. If the point $Q = (-2, 12)$ lies on the parabola. Find $|QF|$ (i.e., the distance between Q and F).

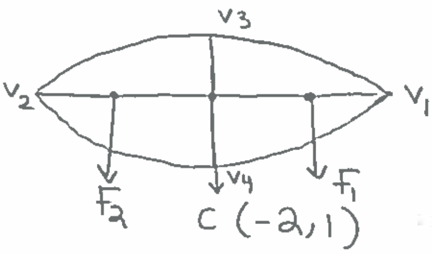


$$|QF| = |QL| = 8$$

QUESTION 12. (6 points) Consider the ellipse $\frac{(y-1)^2}{9} + \frac{(x+2)^2}{25} = 1$.

$9 \to b^2$ $25 \to$ big # so its $\left(\frac{k}{2}\right)^2$ so the shape is

(i) Sketch (roughly)



(ii) Find the foci of the ellipse

$$CF^2 = \left(\frac{k}{2}\right)^2 - b^2 = 25 - 9 = 16$$

$CF^2 = 16$ so $\boxed{CF = 4}$

so $F_1\ (-2+4, 1)$ $(2,1)$

$F_2\ (-2-4, 1)$ $(-6,1)$

(iii) Find all four vertices of the ellipse.

$\left(\frac{k}{2}\right)^2 = 25$ $\boxed{\pm\frac{k}{2}}$

$\boxed{\frac{k}{2} = 5}$ $\boxed{\pm b}$

$b^2 = 9$ $\boxed{b = 3}$

$V_1 = (-2+5, 1)$ $(3,1)$

$V_2 = (-2-5, 1)$ $(-7,1)$

$V_3 = (-2, 1+3)$ $(-2,4)$

$V_4 = (-2, 1-3)$ $(-2,-2)$

QUESTION 13. (4 points) Given $Q = (1, 6, 4)$ is not on the line $L : x = t + 1, y = 2t + 4, z = -5t + 3$ $(t \in R)$. Find $|QL|$.

$D = \langle 1, 2, -5 \rangle$

$I = \langle 1, 4, 3 \rangle$

$IQ = \langle 0, 2, 1 \rangle$ $Q - I$

$$IQ \times D = \begin{vmatrix} i & j & k \\ 0 & 2 & 1 \\ 1 & 2 & -5 \end{vmatrix} = -12i + 1j - 2k$$

$$|QL| = \frac{|D \times IQ|}{|D|} = \frac{\sqrt{12^2 + 1^2 + 2^2}}{\sqrt{1^2 + 2^2 + 5^2}} = \frac{\sqrt{149}}{\sqrt{30}}$$

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