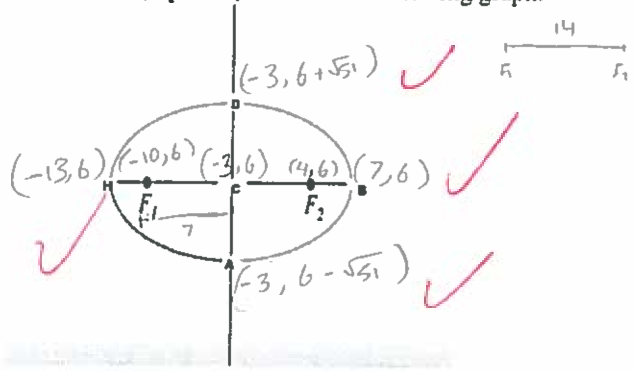
Name Haya Sujaa, ID g00082558

Final Exam, MTH 111, Spring 2019

Ayman Badawi

$$Score = \frac{75}{78}$$

QUESTION 1. (7 points) Stare at the following graph.



(i) Given $F1 = (-10, 6)$, $F2 = (4, 6)$ and the ellipse-constant is 20.

(ii) Find the center $c =$

$|CF1| = 7 \quad \therefore C = (-3, 6)$

(iii) Find the vertices $A = (-3, -1.14)$, $D = (-3, 13.14)$, $H = (-13, 6)$, and $B = (7, 6)$

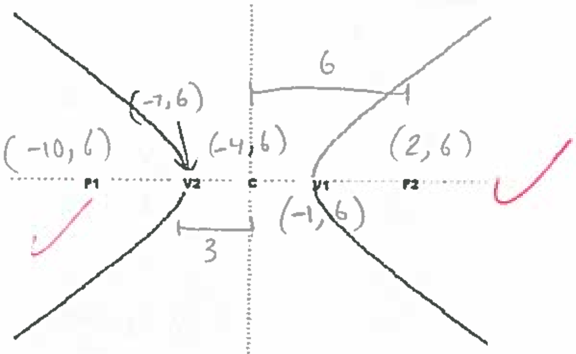
$100 - 49 = b^2$

$b = \sqrt{51}$

(iv) Find the equation of the ellipse.

$$\frac{(x+3)^2}{100} + \frac{(y-6)^2}{51} = 1$$

QUESTION 2. (6 points) Stare at the following graph.



Given $c = (-4, 6)$, $|cv2| = 3$, and $F2 = (2, 6)$.

(i) Find $v1 = (-1, 6)$ $F1 = (-10, 6)$, $v2 = (-7, 6)$, and the hyperbola-constant $k = 6$

$|CF_1| = \sqrt{(6/2)^2 + b^2} = 6$

$\sqrt{9 + b^2} = 6$

$9 + b^2 = 36$

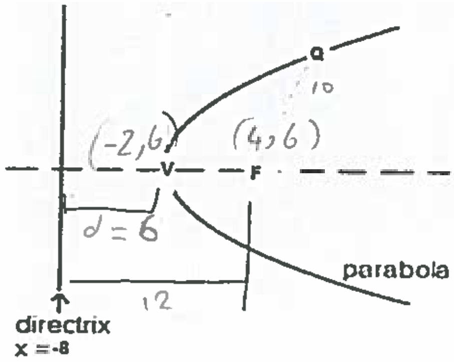
$b^2 = 36 - 9$

$b^2 = 27$

(ii) Find the equation of the hyperbola

$$\frac{(x+4)^2}{9} - \frac{(y-6)^2}{27} = 1$$

QUESTION 3. (4 points) Stare at the following graph.



Given $F = (4, 6)$, the directrix line, L is $x = -8$, and $|QF| = 10$.

(i) Find $|QL| = |QF| = 10$

(ii) Find $v = (-2, 6)$

(iii) Find the equation of the parabola

$$24(x+2) = (y-6)^2$$

QUESTION 4. (6 points). Find y' and do not simplify

(i) $y = ln[(4x+3)^{10}(-5x+30)^3]$

$$y = \ln(4x+3)^{10} + \ln(-5x+30)^3$$
$$y = 10\ln(4x+3) + 3\ln(-5x+30)$$
$$y' = \frac{10 \cdot 4}{4x+3} + \frac{3 \cdot -5}{-5x+30}$$

$$y' = \frac{40}{(4x+3)} + \frac{-15}{(-5x+30)}$$

(ii) $y = e^{(6x^3+x^2-1)} + 10x^2 - x + 23$

$$y = \left[\left[e^{(6x^3+x^2-1)} \cdot (18x^2+2x)\right] + 20x - 1\right]$$

(iii) $y = (21 + 5x - 6x^3)^7$

$$y' = 7(21+5x-6x^3)^6 \cdot (5 - 18x^2)$$

QUESTION 5. (6 points).

(i) Find $\int x e^{(x^2+1)}\, dx$

$u = x^2 + 1$
$u' = 2x$

$$\frac{1}{2}\left(e^{(x^2+1)}\right) + C$$

(ii) Find $\int \frac{e^{2x}+1}{(e^{2x}+2x-5)^3}\, dx$

$$\int (e^{2x}+1)(e^{2x}+2x-5)^{-3}\, dx$$
$u = e^{2x} + 2x - 5$
$u' = 2e^{2x} + 2$
$$\frac{1}{2}\int 2(e^{2x}+1)(e^{2x}+2x-5)^{-3}\, dx$$

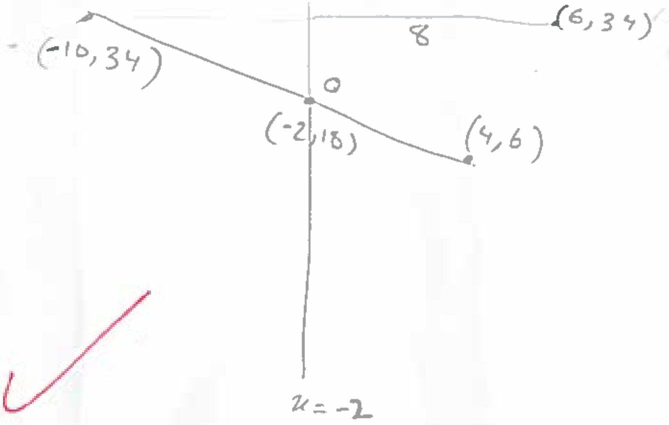
$$\frac{1}{2} \cdot \frac{1}{-2}\left(e^{2x}+2x-5\right)^{-2} + C$$

(iii) Find $\int (6x+3)(x^2+x-5)^{11}\, dx$

$u = x^2 + x - 5$
$u' = 2x + 1$

$$3 \cdot \frac{1}{12}(x^2+x-5)^{12} + C$$

QUESTION 6. (5 points). Let $H = (4,6)$, $F = (6,34)$. Find a point Q on the line $x = -2$ such that $|HQ| + |FQ|$ is minimum.



$y = mx + b$

$m = \frac{6-34}{4+10} = -2$

$6 = -2(4) + b$

$b = 14$

$y = -2x + 14$

$y = -2(-2) + 14$

$= 18$

$Q = (-2, 18)$

QUESTION 7. (4 points). For what values of x does the tangent line to the curve $y = ln(4x+1) + 7x + 2$ have slope equal 8?

$y' = 8$

$y' = \frac{4}{4x+1} + 7 = 8$

$\frac{4}{4x+1} = 1$

$4 = 4x + 1$

$4x = 4 - 1$

$x = 3/4$

check $\frac{4}{4(\frac{3}{4})+1} + 7 =$

$1 + 7 = 8$ ✓

the T line has slope 8 at $x = \frac{3}{4}$

QUESTION 8. (6 points). The plane $P_1 : x + 2y - 3z = 2$ intersects the plane $P_2 : -x + 5y + z = 19$ in a line L. Find a parametric equations of L.

(1) → $N_1 \times N_2 = D$

$N_1 = \langle 1, 2, -3 \rangle$

$N_2 = \langle -1, 5, 1 \rangle$

$D = (2+15)i - (1-3)j + (5+2)k$

$= \langle 17, 2, 7 \rangle$

(2) → $z = 0$

$x + 2y = 2$

$-x + 5y = 19$

$x = \frac{\begin{vmatrix} 2 & 2 \\ 19 & 5 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$ $\quad y = \frac{\begin{vmatrix} 1 & 2 \\ -1 & 19 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ -1 & 5 \end{vmatrix}}$

$x = \frac{-28}{7}$ $\quad y = \frac{21}{7}$

$x = -4$ $\quad y = 3$

(3) → $(-4, 3, 0)$

$D = \langle 17, 2, 7 \rangle$

$L: \left.\begin{array}{l} x = 17t - 4 \\ y = 2t + 3 \\ z = 7t \end{array}\right\} t \in \mathbb{R}$

QUESTION 9. (5 points). Can we draw the entire line $L : x = 2t, y = -3t + 1, z = 11t + 4$ inside the plane $2x - 6y - 2z = 20$? EXPLAIN

$N_{plane} \cdot D_{line}$ must $= 0$

$N = \langle 2, -6, -2 \rangle$

$D = \langle 2, -3, 11 \rangle$

$N \cdot D = 4 + 18 - 22$

$= 0$ ✓

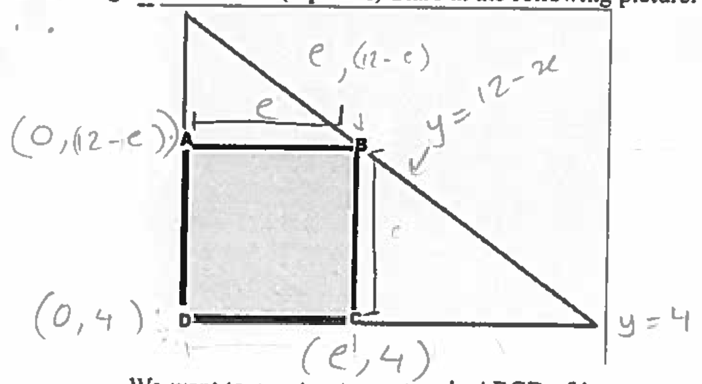
NO

yes the line can be entirely drawn on the plane because the dot product of the normal and directional vector is 0

take a point on L and check if the point lies in the plane or not

3

QUESTION 10. (8 points) Stare at the following picture.



We want to construct a rectangle ABCD of largest area as in the picture above. Note that A and D lie on the y-axis, D and C lie on the line y = 4 (note that y = 4 intersects the y-axis at D), and B lies on the line y = 12 - x. Find |DC| and |BC|.

$|BC| = (12 - e) - 4$

$|DC| = e$

$A = |BC| \cdot |DC|$

$= [(12 - e) - 4] \cdot e$

$= (-e + 8)e$

$= -e^2 + 8e$

$A' = -2e + 8$

① → $-2e + 8 = 0$

$e = 4$

② → $|BC| = (12 - 4) - 4$

$= 8 - 4$

$= 4$ units

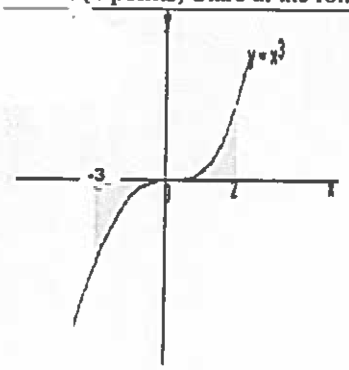
$|DC| = e$

$= 4$ units

Area $= 4 \times 4$

$= 16$ units^2

QUESTION 11. (4 points) Stare at the following picture.



Find the area of the shaded region. Note that $y = f(x) = x^3$ and x is between -3 and 2.

$$A = \left[\int_{-3}^{0} x^3\,dx\right] + \int_{0}^{2} x^3\,dx$$

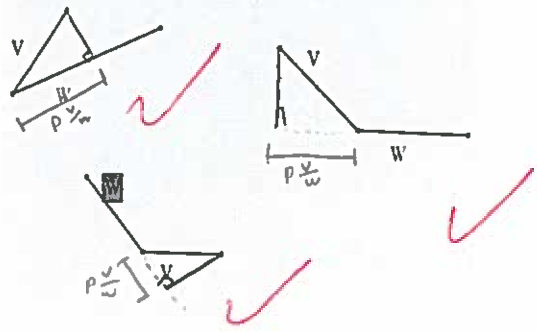
$$= \left[\int_{-3}^{0} \frac{1}{4}x^4\right] + \int_{0}^{2} \frac{1}{4}x^4$$

$$= \left[\left[\frac{1}{4}0^4\right] - \left[\frac{1}{4}(-3)^4\right]\right] + \left[\left[\frac{1}{4}(2)^4\right] - \left[\frac{1}{4}(0)^4\right]\right]$$

$$= [0 + 20.25] + [4 - 0]$$

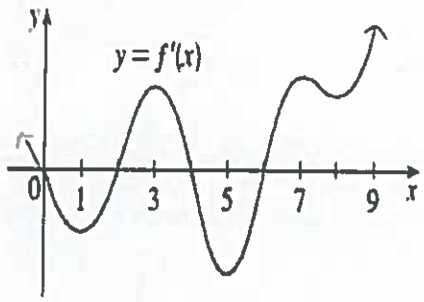
$$= 24.25 \text{ units}^2$$

QUESTION 12. (4.5 points) Stare at the following picture.



Draw the projection of V over W.

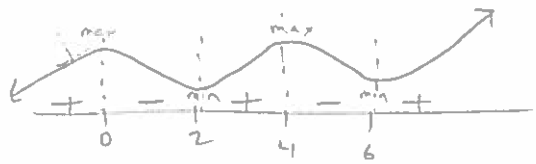
QUESTION 13. (7.5 points) Stare at the following graph of $y = f'(x)$.



critical values $= 0, 2, 4, 6$

$\nearrow = (-\infty, 0), (2, 4), (6, +\infty)$

$\searrow = (0, 2), (4, 6)$



(i) At what value(s) of x does $f(x)$ have local max.?

at $x = 0$ and $x = 4$

(ii) At what value(s) of x does $f(x)$ have local min.?

at $x = 2$ and $x = 6$

(iii) For what values of x does $f(x)$ increase?

$(-\infty, 0) \cup (2, 4) \cup (6, +\infty)$

(iv) For what values of x does $f(x)$ decrease?

$(0, 2) \cup (4, 6)$

(v) For what values of x will the normal lines have positive slope.

Normal line will have a + slope the the tangent line has − slope

∴ when the function x is decreasing : $(0, 2) \cup (4, 6)$

QUESTION 14. (5 points) Given $L_1 : x = 2t, y = t + 1, z = 3t$ is perpendicular to $L_2 : x = 4w + 6, y = -2w, z = aw + 1$ and they intersect at a point Q. Find the value of a and find the point Q.

$L_1 : \left.\begin{array}{l} x = 2t \\ y = t+1 \\ z = 3t \end{array}\right\} t \in \mathbb{R}$ $\quad L_2 : \left.\begin{array}{l} x = 4w+6 \\ y = -2w \\ z = aw+1 \end{array}\right\} w \in \mathbb{R}$

$Q = (2, 2, 3)$

$a = -2$

$x = x \quad y = y$

$2t = 4w + 6 \quad | \; t + 1 = -2w$

$\quad | \; t + 2w = -1$

$2t - 4w = 6$

$t + 2w = -1$

$t = \dfrac{\begin{vmatrix} 6 & -4 \\ -1 & 2 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}} \qquad w \dfrac{\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix}}{\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}} \qquad t = 1 \quad w = -1$

$x = 2$
$y = 2$
$z = 3$

$z = aw + 1$
$3 = a(-1) + 1$
$3 - 1 = a(-1)$
$2 = a \cdot -1$
$a = \frac{2}{-1} \quad a = -2$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com