Name————————————————————, **ID** ————————————

Exam One, MTH 213 , Fall 2021

Ayman Badawi

(**Stop working at 14:45 pm/ submit your solution by 15:00 pm**) _____
36

QUESTION 1. (**12 points**)(**SHOW THE WORK**)

(i) Find $\ell\{e^{4t}cos(5t)\}$

(ii) Find $\ell\{U_2(t)e^{(7t-14)}sin(t-2)\}$

(iii) Find $\ell^{-1}\{\frac{s}{(s+7)^3}\}$

(iv) Find $\ell^{-1}\{\frac{e^{-4s}}{s^2-9}\}$

QUESTION 2. (**SHOW THE WORK**)(**6 points**)
Solve $y^{(2)} - 5y' + 6y = 6$, such that $y(0) = y'(0) = 0$.

QUESTION 3. (**SHOW THE WORK**)(**6 points**) Solve $y^{(2)} + 10y' + 34y = 0$, such that $y(0) = 1, y'(0) = 7$.

QUESTION 4. (**SHOW THE WORK**)(**6 points**) Solve $y' - 3y = U_2(t)$, such that $y(0) = 0$

QUESTION 5. (**SHOW THE WORK**)(**6 points**) Solve $y^{(2)} - 4y' = 1$, such that $y(0) = y'(0) = 0$

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Q1

(i) $\mathcal{L}\{e^{4t}\cos(5t)\}$

$$\mathcal{L}\{\cos(5t)\} = \frac{s}{s^2+25}$$

$$\mathcal{L}\{e^{4t}\cos(5t)\} = \boxed{\frac{(s-4)}{(s-4)^2+25}}$$

(ii) $\mathcal{L}\{u_2 \cdot e^{7t-14} \cdot \sin(t-2)\}$

$\rightarrow e^{-2s}\mathcal{L}\{e^{7(t+2)-14} \cdot \sin(t+2-2)\}$

$\rightarrow e^{-2s}\mathcal{L}\{e^{7t} \cdot \sin t\}$

$\rightarrow \boxed{e^{-2s} \cdot \frac{1}{(s-7)^2+1}}$

(iii) $\mathcal{L}^{-1}\left\{\frac{s}{(s+7)^3}\right\}$

$\mathcal{L}^{-1}\left\{\frac{s+7-7}{(s+7)^3}\right\} \rightarrow \mathcal{L}^{-1}\left\{\frac{s+7}{(s+7)^3}\right\} - 7\mathcal{L}^{-1}\left\{\frac{1}{(s+7)^3}\right\}$

$\rightarrow \mathcal{L}^{-1}\left\{\frac{1}{(s+7)^2}\right\} - \frac{7}{2!}\mathcal{L}^{-1}\left\{\frac{2!}{(s+7)^3}\right\}$

$\rightarrow t \cdot e^{-7t} - \frac{7}{2}t^2 e^{-7t}$

$\rightarrow \boxed{e^{-7t}\left(t - \frac{7}{2}t^2\right)}$

(iv) $\mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s^2-9}\right\} = u_4\,\mathcal{L}^{-1}\left\{\frac{1}{s^2-9}\right\}$

$\rightarrow \frac{u_4}{3}\sinh(3t)$

$\rightarrow \boxed{\frac{u_4}{3}\sinh(3(t-4))}$

✓ 3

Q2

solve $y'' - 5y' + 6y = 6$ given $y(0) = y'(0) = 0$

$\mathcal{L}\{y'' - 5y' + 6y\} = \mathcal{L}\{6\}$

$s^2Y(s) - \cancel{sy(0)} - \cancel{y'(0)} - 5sY(s) + \cancel{5y(0)} + 6Y(s) = \frac{6}{s}$

$Y(s)[s^2 - 5s + 6] = \frac{6}{s}$

$Y(s)[(s-2)(s-3)] = \frac{6}{s}$

$Y(s) = \frac{6}{s(s-2)(s-3)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-3}$

using cover method → $A = \frac{6}{6} = 1$ $\quad B = \frac{6}{-2} = -3$ $\quad C = \frac{6}{3} = 2$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{3}{s-2} + \frac{2}{s-3}\right\}$

$\boxed{y(t) = 1 - 3e^{2t} + 2e^{3t}}$

✓ 6

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Q3

solve $y'' + 10y' + 34y = 0$ given $y(0) = 1$, $y'(0) = 7$

$$\mathcal{L}\{y'' + 10y' + 34y\} = \mathcal{L}\{0\}$$

$$s^2Y(s) - sy(0) - y'(0) + 10sY(s) - 10y(0) + 34Y(s) = 0$$

$$s^2Y(s) + 10sY(s) + 34Y(s) - s - 7 - 10 = 0$$

$$Y(s)[s^2 + 10s + 34] = s + 17$$

$$Y(s)\left[(s + \tfrac{10}{2})^2 - (\tfrac{10}{2})^2 + 34\right] = s + 17$$

$$Y(s)[(s+5)^2 + 9] = s + 17$$

$$Y(s) = \frac{s+17}{(s+5)^2 + 9}$$

$$Y(s) = \frac{s+5}{(s+5)^2 + 9} + \frac{12}{(s+5)^2 + 9}$$

$$\mathcal{L}^{-1}(Y(s)) = \mathcal{L}^{-1}\left\{\frac{s+5}{(s+5)^2+9}\right\} + \frac{12}{3}\mathcal{L}^{-1}\left\{\frac{3}{(s+5)^2+9}\right\}$$

$$y(t) = e^{-5t} \cdot \cos 3t + 4\sin 3t \cdot e^{-5t}$$

$$\boxed{y(t) = e^{-5t}(\cos 3t + 4\sin 3t)}$$

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Q4

solve $y' - 3y = u_2$ given $y(0) = 0$

$\mathcal{L}\{y' - 3y\} = \mathcal{L}\{u_2\}$

$sY(s) - y(0) - 3Y(s) = \frac{e^{-2s}}{s}$

$Y(s)[s-3] = \frac{e^{-2s}}{s}$

$Y(s) = e^{-2s} \cdot \frac{1}{s(s-3)}$

$Y(s) = e^{-2s}\left[\frac{A}{s} + \frac{B}{s-3}\right]$ → using cover method

$A = -\frac{1}{3}$ $B = \frac{1}{3}$

$Y(s) = e^{-2s}\left[\frac{1}{3}\cdot\frac{1}{s-3} - \frac{1}{3}\cdot\frac{1}{s}\right]$

$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{e^{-2s}\left[\frac{1}{3}\cdot\frac{1}{s-3} - \frac{1}{3}\cdot\frac{1}{s}\right]\}$

$y(t) = u_2\left(\frac{1}{3}e^{3(t-2)} - \frac{1}{3}\right)$

$$\boxed{y(t) = \frac{u_2}{3}\left(e^{3(t-2)} - 1\right)}$$

6

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Q5

solve $y'' - 4y' = 1$ given $y(0) = y'(0) = 0$

$$s^2Y(s) - \cancel{sy(0)} - \cancel{y'(0)} - 4sY(s) + \cancel{4y(0)} = \frac{1}{s}$$

$$Y(s)[s^2 - 4s] = \frac{1}{s}$$

$$Y(s)[s(s-4)] = \frac{1}{s}$$

$$Y(s) = \frac{1}{s^2(s-4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-4}$$

by cover method $\rightarrow B = -\frac{1}{4}$ $C = \frac{1}{16}$

$$A \rightarrow As^2 - 4As + Bs - 4B + Cs^2 = 1$$

$$A + C = 0 \rightarrow A + \frac{1}{16} = 0 \rightarrow A = -\frac{1}{16}$$

$$Y(s) = -\frac{1}{16} \cdot \frac{1}{s} - \frac{1}{4} \cdot \frac{1}{s^2} + \frac{1}{16} \frac{1}{s-4}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\{-\frac{1}{16} \cdot \frac{1}{s}\} - \frac{1}{4}\mathcal{L}^{-1}\{\frac{1}{s^2}\} + \frac{1}{16}\mathcal{L}^{-1}\{\frac{1}{s-4}\}$$

$$\boxed{y(t) = -\frac{1}{16} - \frac{1}{4}t + \frac{1}{16}e^{4t}}$$